Nonlinear dynamics of a microcantilever in close proximity to a surface

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***Abstract* — We compute the Correlation Dimension, the Bifurcation Diagram, and a Poincaré Map for experimental data obtained by dynamic atomic force microscope. From the analysis we infer a period doubling mode and chaotic modes of the system.**

***Index Terms* — Nanotechnology, atomic force microscopy, nonlinear oscillators, chaos, microelectromechanical devices.**

I. Introduction

Atomic-force microscopy (AFM) is a widely used tool for surface analysis today. Dynamic AFM methods like tapping mode™ or noncontact mode are commonly used for imaging although a thorough understanding of the data can be difficult due to the nonlinear tip–sample interaction [1]. In the tapping-mode the micro cantilever is resonantly forced to oscillations with an amplitude of a about ten to one hundred nanometers. Close to the specimen the tip periodically interacts with the surface which reduces the oscillation amplitude. From theoretical investigations it is known that the nonlinear interaction with the specimen can lead to complex dynamics although the system is well behaved for a large set of parameters [2-5]. The possibility of chaotic motion was shown using the Melnikov method [6,7]. Double [8] and higher period orbits [3] were predicted, too. The nonlinear tip–sample interaction also leads to the generation of higher harmonics. The occurrence of higher harmonic and subharmonic signals has been analyzed theoretically employing a multiple-degree-of-freedom (MDOF) model [9]. These harmonic signals allow one to reconstruct the transient tip–sample interaction forces [10]. In this contribution we analyze the nonlinear dynamics of tapping mode™ AFM.

II. Experimental

A Multimode Nanoscope IIIa (Veeco, Santa Barbara, CA) atomic force microscope was used with external amplification of the deflection signal (SR560, Stanford Research, Sunnyvale, CA) as illustrated in Fig. 1. Data were recorded at 5 MS/s (NI PCI-6110E, National Instruments, Austin, TX). The microscope was operated in tapping mode™ with a free amplitude of 45 nm. The spring constant 3.8 N/m of the v-shaped Si-cantilever (200 µm length; type NSCH 11, Silicon-MDT, Moscow) was determined by standard thermal-noise calibration. The experiments were carried out on a silicon (100) waver with natural oxide layer. The sample was cleaned with ethanol and ultrapure water before use.

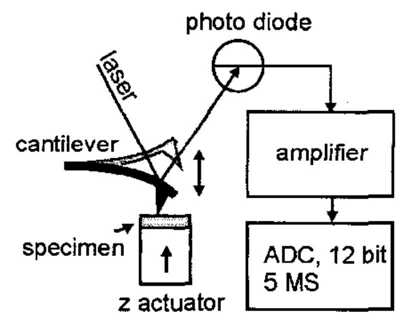

Fig. 1. Experimental setup. The tip of the atomic force microscope was driven at its resonance frequency.

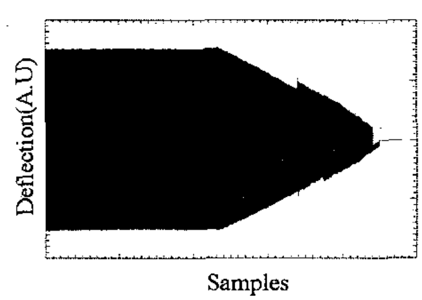

Fig. 2. Raw data of the photo diode signal versus time. The amplitude reduction due to the approaching sample and the discontinuity due to the transition between the attractive regime to the repulsive regime are evident.

III. Data Analysis

Experimental data were analyzed using methods from the theory of nonlinear systems. For that purpose the data has been embedded into a higher dimensional state space using the method of delay coordinates [11,12,13]. The dimension of the state space was set to three. The mutual information of the time series with a shifted copy of itself has been minimized in order to find the optimal embedding delay, which was found to be T=108.2235 samples ($\omega_0 = 2\pi/T$). In Fig. 3 delay coordinate embeddings of the time series are shown for eight different sub-regions of the total data. Far away from the surface the tip is in a quasi-free state of motion which is reflected by an almost perfect circular orbit in state space. During the approach of the surface the perfect orbit becomes more and more distorted until the attractive state becomes unstable and the system transits into the repulsive state which is accompanied by a phase jump. After further approach the trajectory shows a period-doubling and finally a transition into a fully chaotic motion. The chaotic regime is interrupted by regions of quasi-regular behavior. Fig. 3 shows a plot of the correlation dimension D2 for different times during the approach. The correlation dimension here serves as a measure for the fuzzyness of the trajectory in state space. Smaller values in the range near one correspond to quasi-regular motion while values near two correspond to more chaotic behavior and values near three correspond to an erratic chaotic motion of the system.

A bifurcation diagram (Fig. 4) has been computed by using a stroboscopic sampling with a period of T=108.2235 samples, where non-integer values have been interpolated using a 4th order scheme. One can clearly see the initial regular motion of the system, the attractive mode, the transition to the repulsive mode, a period-doubling of the system and a transition to a chaotic motion very close to the surface.

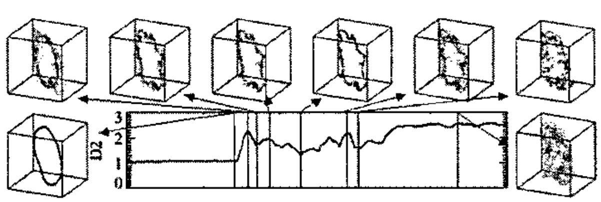

Fig. 3: 3-D phase space embeddings for eight different times together with the correlation dimension D2. The time window is the same as in the main panel of Fig. 4. The arrows indicate the time of the respective return plot. The first panel shows the return plot in the regular region. The second panel is located at the phase jump from the attractive to the repulsive mode. The fifth panel shows the period doubled mode. The seventh panel shows a period four mode and the last panel shows the chaotic regime.

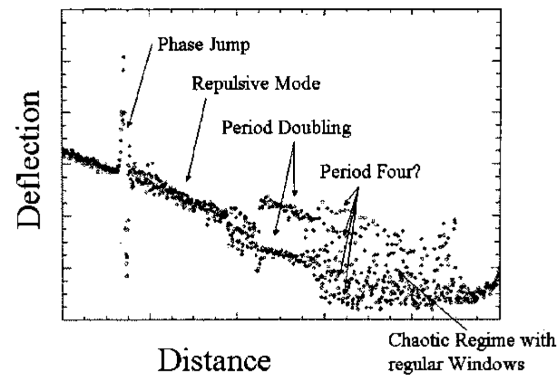

Fig. 4. Bifurcation diagram for the experimentally recorded AFM data. It shows a close-up for the transition from regular to chaotic behavior in the range of small tip-sample distances. After a transition from the attractive to the repulsive mode the system undergoes a period-doubling and finally ends in a chaotic regime.

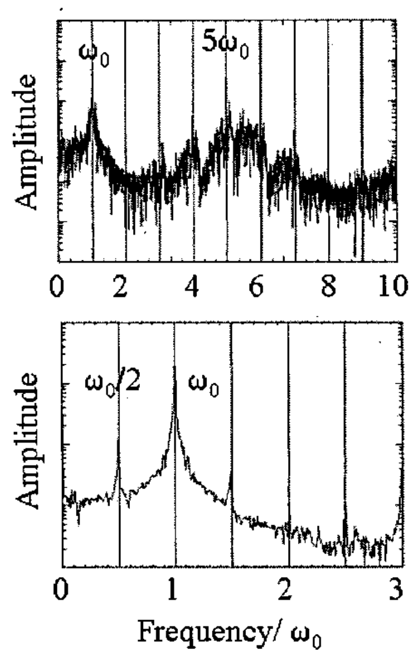

Fig. 5: Fourier transform of the photodiode signal in the chaotic regime and in the period doubling regime.

The period-doubling mode can be further observed in the Fourier spectra of the data in that regime (Fig. 5). For this purpose Fourier transforms of the data have been computed in the period-doubling regime and the fully chaotic regime which reveal strong peaks at multiple frequencies of the excitation frequency up to a multiple of nine times the excitation frequency.

The overall dynamics of the system can be analyzed further by using the method of Poincaré plots as shown in Fig. 6. The signal is sub-sampled at a rate of T/6 which corresponds to six samples per period. They are represented in a delay coordinate state space. Essentially the shown graph is a superposition of six Poincaré plots for different phase shifts of the signal.

The six dark spots on the outer diameter correspond to the regular regime and to the approach phase. The system then undergoes a phase jump which leads to the six-fold structure in the center. Close to the center the period-doubling regime can be observed which leads finally to the chaotic regime of the system. The Poincaré plot further shows clearly the low-dimensionality of the system.

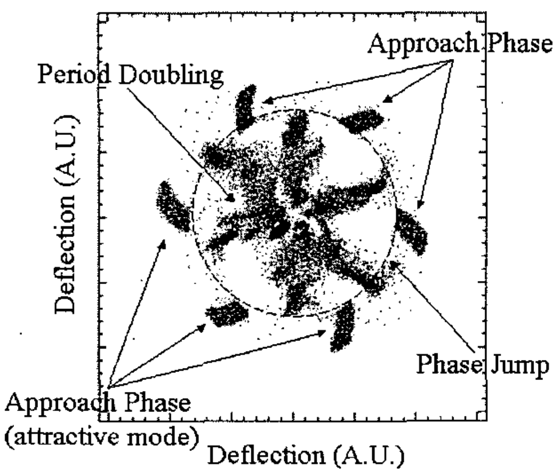

Fig. 6: Poincaré plot for a period of T/6, showing the different regimes of the system.

IV. CONCLUSION

The system under consideration shows features that are typical for a dynamical system with a low number of degrees of freedom in the regular and chaotic regime. Far away from the surface the system is in a regular regime, while during the approach a phase jump, a period-doubling regime and a chaotic regime develop. System characteristics of this type have been observed for the Duffing-oscillator [13] which can serve as a model system for an AFM.

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REFERENCES

[1] R. García and R. Pérez, "Dynamic atomic force microscopy methods," *Surf. Sci. Rep.*, vol. 47, pp. 197-301, 2002.

[2] J. P. Hunt and D. Sarid, "Kinetics of lossy grazing impact oscillators," *Appl. Phys. Lett.*, vol. 72, pp. 2969-2971, 1998.

[3] M. Basso, L. Giarré, M. Dahleh, and I. Mezic, "Complex dynamics in a harmonically excited lennard-jones oscillator: microcantilever-sample interaction in scanning probe microscopes", *J. Dynamic Sys. Meas. Control*, vol. 122, pp. 240-245, 2000.

[4] A. Sebastian et al., "Harmonic and power balance tools for tapping-mode atomic force microscope", *J. Appl. Phys.*, vol. 89, iss. 11, pp. 6473-6480, 2001.

[5] S. Rützel, S. I. Lee and A. Raman, "Nonlinear dynamics of atomic-force-microscope probes in Lennard-Jones Potentials", *Proc. R. Soc. London A*, vol. 459 pp. 1925-1948, 2003.

[6] M. Ashhab, M. Salapaka, M. Dahleh and I. Mezic, "Dynamical analysis and control of microcantilevers", *Automatica* vol. 35, no. 10, pp. 1663-1670, 1999.

[7] M. Ashhab, M. V. Salapaka, M. Dahleh, and I. Mezic, "Melnikov-based dynamical analysis of microcantilevers in scanning probe microscopy", *Nonlinear Dynamics*, vol. 20, pp. 197-220, 1999.

[8] N. Sasaki, M. Tsukada, M. Tamura, R. Tamura, K. Abe, and N. Sato, "Dynamics of the cantilever in noncontact atomic-force microscopy", *Appl. Phys. A*, vol. 66, pp. S287-S291, 1998.

[9] R. W. Stark, "Spectroscopy of higher harmonics in dynamic atomic force microscopy", *Nanotechnology*, vol. 15, pp. 347-351, 2004.

[10] M. Stark, R. W. Stark, W. M. Heckl and R. Guckenberger, "Inverting dynamic force microscopy: From signals to time-resolved interaction forces", *Proc. Natl. Acad. Sci. USA*, vol. 99, p. 8473, 2002.

[11] F. Takens, "Detecting Strange Attractors in Turbulence", *Lecture Notes in Math. vol. 898*, Springer, New York, 1981.

[12] T. Sauer, J. Yorke, and M. Casdagli, "Embedology", *J. Stat. Phys.*, vol. 65, p. 579, 1991.

[13] F. Battelli, K. J. Palmer. "Chaos in the Duffing equation", *J. Diff. Eqns.*, vol. 101, pp. 276–301, 1993.

[14] N. H. Packard, J. P. Cratchfield, J. D. Farmer, R. S. Shaw, *Phys. Rev. Lett.* vol. 45, p. 712, 1980.